دراسة عددية للمسألة العكسية ذات القيم الحدودية غير المنتظمة النطاق باستخدام اساس السبلاين التكعيبي

Numerical study of Inverse Problem For Boundary value problem Using B - Cubic Spline

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الملخص

فى هذا البحث قدم الحل العددى للمسألة العكسية المتضمنة مسالة قيم حدودية ذات نطاق غير منتظم وباستخدام أساس السبلاين التكعيبى. كذلك درست تمييز نطاق التعريف للمسألة.

Abstract:

In this paper, a numerical solution for inverse problem involving Irregular boundary value problem using B -cubic spline has been presented.

Also identification of the domain of definition is studied.

I. Introduction

The inverse problems are mathematical problems appears in many applications of mathematics such that some information about the solution of the problem is given

and some parameters (or conditions functions) are unknown in this problem .

The inverse problem is used in various area, such dynamics ,engineering, statistics, numerical analysis, mathematical modeling, etc [7] , [1] . In this paper ,the B-cubic spline base has been used in approximation .

2.Preliminaries

2.1 B - Cubic Spline Base

We first partition the interval [-2,2] by choosing the length of partition equal to 1, [6] . This produce the equally space nodes $x_i = -2 + i$ **for each i = 01,2,3,4 .**

Define $B : [-2,2] \longrightarrow [0,1]$ **as B-cubic spline function on the interval [-2, 2] satisfy the smoothness properties i.e.** $B \in C^2(-\infty, \infty)$ **such that :**

$$B(x) = \begin{cases} (2-x)^3 - 4(1-x)^3 - 6x^3 + 4(1+x)^3 & -2 \le x < -1 \\ (2-x)^3 - 4(1-x)^3 - 6x^3 & -1 \le x < 0 \\ (2-x)^3 - 4(1-x)^3 & 0 \le x < 1 \\ (2-x)^3 & 1 \le x \le 2 \end{cases}$$

Also the B Cubic Spline Base $B = \{B_0(x), B_1(x), B_2(x), ,..., B_n(x)\}$, **[5] in the interval** $[a, b]$ **which has the length of partition** $h = \frac{b-a}{n}$ **with** nodes of partition $x_i = a + i * h)$ **for i=0,1,2,3,…,n have been define as following :**

$$B_i(x) = B\left(\frac{x - x_i}{h}\right)$$ **for i=0,1,2,…,n , [5] .**

2.2 The method of solution

Consider the Boundary value problem (B.V.P.)

$$\left.\begin{aligned} Lu &= f(x, y) \quad && in \quad R \\ u &= 0 && on \quad \partial R \end{aligned}\right\} \qquad (2.1)$$

Such that

$$R = \{(x, y) : a \le x \le b, 0 \le y \le h(x)\}$$

See figure 1, problem 2.1 has unique solution if h(x) is given . Suppose that this problem has solution $u^*(x,y)$ and the function h(x) is unknown.

Now to estimate h(x), it will be approximated by linear combination of Cubic spline functions base $B_0(x), B_1(x), B_2(x), ,..., B_n(x)$ which are linearly independent and smoothness .

Let $h^*(x) = \sum_{k=1}^{n} a_k B_k(x)$

In this paper, the least squares method is used to estimate the parameters $a_1, a_2,,,,, a_n$ from

$$J(a_1, a_2,..., a_n) = \sum_i \sum_j \left(u(x_j, y_i, \vec{a}) - u^*(x_j, y_i) \right)^2$$

Where

u^* **represents the given exact solution.**

u **represents the Numerical solution of problem.**

$\vec{a} = (a_1, a_2,,,,, a_n)$ **the vector of the unknown parameters**

The values of parameters can be found by minimizing J with respect to a_i for i=1,2,…,n

The method solution of inverse problem will be transform into the unconstrained nonlinear programming problem $\min J(\vec{a})$

There are many methods to solve the problem (2.1), the variational method has been used, [6] also the B-cubic

spline base has been used to approximate the solution of boundary value problem (2.1) .

2.1.1 The Variational Method

Let the boundary value problem :

$\nabla^2 u = f$ in R

$u = 0$ on ∂R

Such that

$$R = \{(x,\ y) : a \le x \le b\ ,\ c \le y \le h(x)\ \}$$

The equivalent functional to this problem is :

$$J(u) = \int_a^b \int_0^{h(x)} (u_x^2 + u_y^2 + 2uf\ )dxdy \qquad (2.2)$$

[6] . Define the norm of x-interval as follow : $h = \dfrac{b-a}{n}$

And the norm of y-interval is : $k_x = \dfrac{h(x)}{m}$ **such that n and m are two positive integer numbers [4] .**

The B cubic spline base in x has been used such that satisfy the boundary conditions of problem . It means that :

$B_i(a,h) = B_i(b,h) = 0$ **for i=0,1,2,..,m**

Also, the B -cubic spline base in y has been used such that satisfy the boundary conditions of problem . It means that :

$B_i(h(x),k_x) = B_i(0,k_x) = 0$ **for i=0,1,2,…,n**

So that . the solution is

$$u(x,y) = \sum_{i,j=0}^{N,M} \alpha_{ij}\ B_i(x,h) B_j(y,k_x) \qquad (2.3)$$

Which Satisfy B.C. of problem [3] .

Substitute equation (2.3)in the equation (2.2) and use partial derivative with respect to parameters z_{ij} that mean :

$$\frac{\partial J}{\partial z_{kl}} = 0 \quad for \quad k = 0,1,2,..., n \quad ,l = 0,1,2,..., \quad m$$

The Guassin method is used to solve the system of linear equations Az=b .

To solve the unconstrained nonlinear programming .The Hooke and Jeeves method is used, Results are given in figure (2) which illustrate the results in case
h(x)=1+x(x-1)
u(x,y)=xy(1-x)(1+x(1-x)-y)

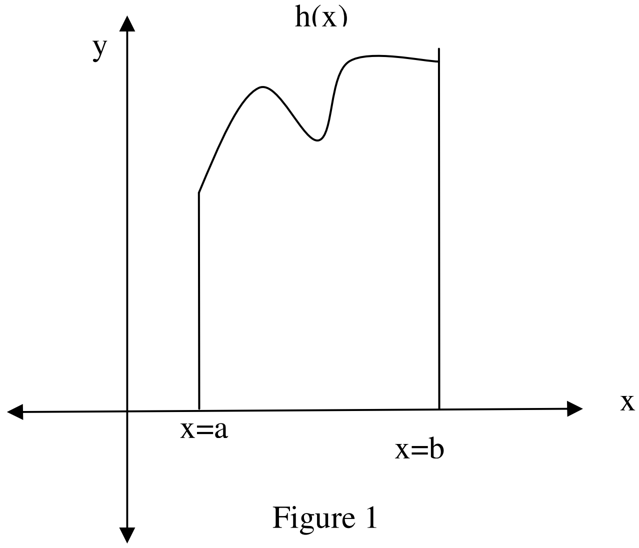

Figure 1

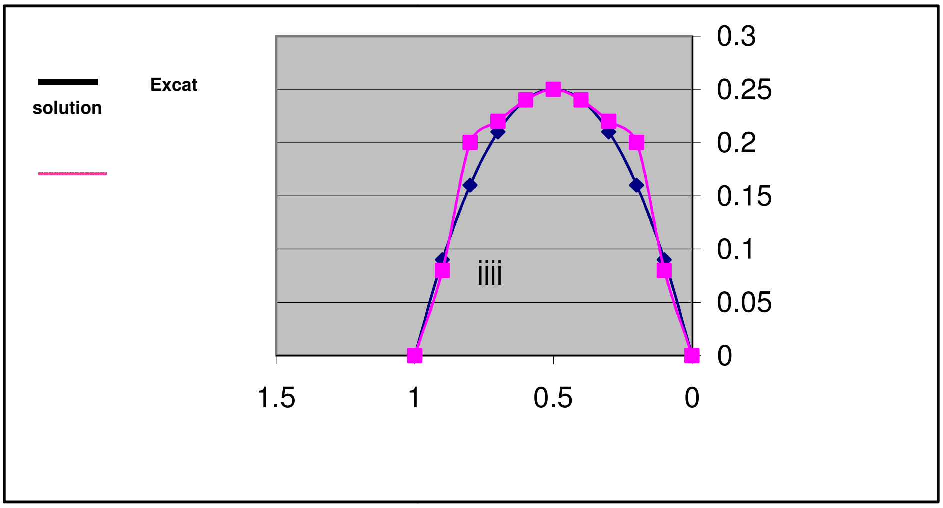

Figure 2

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