

الفضاءات التبولوجية الثلاثية

عند بارا – ليندوف و سيمبيرا – ليندوف

On Para-Lindelöf and Semipara-Lindelöf
Tritopological Spaces

المدرس المساعد

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Abstract:

We define a para-Lindelöf tritopological Space and Semipara-Lindelöf tritopological space and we find some properties of these concepts and give the relation between these concepts .

Introduction:

The concept of paracompactness is due to Dieudonne [4].The concept of para- Lindelöf is due to Fleissner [5].The term space (X, τ, ρ) is referred to as a set X with two general non identical topologies τ and ρ [2].The term space (X, τ, ρ, μ) is referred to as as set X with three general non identical topologies τ , ρ and μ [3].Kovar [3] introduce the concept of paracompactness in the tritopological (3topological) space.Al-Fatlawee [1] study the concept of paracompactness in bitopological spaces and tritopological space and give several results and properties of this concept.A collection of subsets of X is locally finite (resp.locally countable) [5] with respect to the topology ρ if every $x \in X$ has a ρ -neighborhood meeting finitely many (resp.countable many) elements of the collection.A collection has the σ -property [5] if it is the union of countably

many collection with the property .A cover (or covering)of a space (X, τ) [4] is a collection of subset of X whose union is all of X .A τ -open cover of X is a cover consisting of τ -open sets ,and other adjective applying to subsets apply similarly to cover .If Π and Φ are covers of X ,we say Φ refines Π [4] if each number of Φ is contained in some member of Π .Then we say Φ refines (or refinement of) Π .A subset of a topological space (X, τ) is an F_σ with respect to the topology [4] if it is a countable union of τ -closed sets ,and written by $\tau - F_\sigma$.

2- Para-Lindelöf Tritopological Spaces

2.1. Definition [1]

A bitopological space (X, τ, ρ) is said to be $(\tau - \rho)$ -compact (resp. Lindelöf) with respect to ρ if every τ -open cover has a finite (resp. countable) ρ -open subcover.

2.2. Definition [3]

A tritopological space (X, τ, ρ, μ) is said to be $(\tau - \rho)$ -paracompact with respect to μ if every τ -open cover has a ρ -open refinement which is locally finite with respect to μ .

2.3. Definition

A tritopological space (X, τ, ρ, μ) is said to be $(\tau - \rho)$ -para-Lindelöf with respect to μ if every τ -open cover has a ρ -open refinement which is locally countable with respect to μ .

2.4. Proposition:

If a tritopological space (X, τ, ρ, μ) is $(\tau - \rho)$ -Lindelöf with respect to μ then it is $(\tau - \rho)$ -para-Lindelöf with respect to μ .

Proof:

This follows from the fact that every countable collection is locally countable .

2.5.Proposition:

If a tritopological space (X, τ, ρ, μ) is $(\tau - \rho)$ -paracompact with respect to μ then it is $(\tau - \rho)$ -para-Lindelöf with respect to μ

Proof

This follows from the fact that every locally finite collection is locally countable .

2.6.Theorem :

If (X, τ, ρ, μ) is $(\tau - \rho)$ -para-Lindelöf with respect to μ then the τ -closed subspace $(Y, \tau_Y, \rho_Y, \mu_Y)$ is $(\tau_Y - \rho_Y)$ -para-Lindelöf with respect to μ_Y .

Proof

Let $\Phi = \{U_\lambda : \lambda \in \Lambda\}$ be a τ_Y -open cover of Y .Since each U_λ is a τ_Y -open subset of Y ,there is τ -open subset V_λ of X such that $U_\lambda = V_\lambda \cap Y$ for each $\lambda \in \Lambda$.Let $\Pi = \{V_\lambda : \lambda \in \Lambda\} \cup \{X / Y\}$.Then Π is τ -open cover of X .By hypothesis Π has a ρ -open refinement $\Psi = \{W_\gamma : \gamma \in \Gamma\}$ which is locally countable with respect to μ .Set $\Omega = \{W_\gamma \cap Y : \gamma \in \Gamma\}$.Then Ω is ρ_Y -open refinement of Φ which is locally countable with respect to μ_Y .

2.7.Theorem:

Let (X, τ, ρ, μ) be a tritopological space ,and let $\Sigma = \{X_i : X_i \in \tau \cap \rho \cap \mu\}$ be a partition of X .The space (X, τ, ρ, μ) is $(\tau - \rho)$ -para-Lindelöf with respect to μ if and only if the space $(X, \tau_i, \rho_i, \mu_i)$ is $(\tau_i - \rho_i)$ -para-Lindelöf with respect to μ_i .

Proof

The "only if part ",since $X_i = X / \bigcup_{i \neq j} X_j$ is τ -closed then the subspace $(X, \tau_i, \rho_i, \mu_i)$ is $(\tau_i - \rho_i)$ -para-Lindelöf with respect to μ_i for every i by Theorem 2.6.

The " if part ".Let $\Phi = \{U_\lambda : \lambda \in \Lambda\}$ be a τ -open cover of X .The collection $\Pi = \{U_\lambda \cap X_i : \lambda \in \Lambda\}$ be a τ_i -open cover of X_i with cardinality $\leq m$ every i .By hypothesis Π has a ρ_i -open refinement $\Psi_i = \{W_{i\lambda} : \lambda \in \Lambda\}$ which is locally countable with respect to μ_i .Let $\Omega = \left\{ \bigcup_{i \in I} W_{i\lambda} : \lambda \in \Lambda \right\}$.Then Ω is ρ -open refinement of Φ which is locally countable with respect to μ .

2.8.Theorem:

If each τ -open set in a $(\tau - \rho)$ -para-Lindelöf with respect to μ space (X, τ, ρ, μ) is $(\tau - \rho)$ -para-Lindelöf with respect to μ , then every subspace $(Y, \tau_Y, \rho_Y, \mu_Y)$ is $(\tau_Y - \rho_Y)$ -para-Lindelöf with respect to μ_Y .

Proof

Let $\Phi = \{U_\lambda : \lambda \in \Lambda\}$ be a τ_Y -open cover of Y .Since each U_λ is a τ_Y -open subset of Y , there is a τ -open subset V_λ of X such that $U_\lambda = V_\lambda \cap Y$ for each $\lambda \in \Lambda$.Then $G = \bigcup_{\lambda \in \Lambda} V_\lambda$ is a τ -open set.

Let $\Omega = \{V_\lambda : \lambda \in \Lambda\}$ be a τ -open cover of G .By hypothesis G is $(\tau - \rho)$ -para-Lindelöf with respect to μ , thus Ω has a ρ -open refinement $\Psi = \{W_\gamma : \gamma \in \Gamma\}$ which is locally countable with respect to μ .Set $\Sigma = \{W_\gamma \cap Y : \gamma \in \Gamma\}$.Then Σ is ρ_Y -open refinement of Φ which is locally countable with respect to μ_Y .

3. Semipara-Lindelöf Tritopological Spaces

3.1. Definition [3]

A tritopological space (X, τ, ρ, μ) is said to be $(\tau - \rho)$ -semiparacompact with respect to μ , if each τ -open cover of X has a τ -open refinement which is σ -locally finite with respect to μ .

3.2. Definition

A tritopological space (X, τ, ρ, μ) is said to be $(\tau - \rho)$ -semipara-Lindelöf with respect to μ , if each τ -open cover of X has a ρ -open refinement which is σ -locally countable with respect to μ .

3.3. Proposition

If a tritopological space (X, τ, ρ, μ) is $(\tau - \rho)$ -para-Lindelöf with respect to μ then it is $(\tau - \rho)$ -semipara-Lindelöf with respect to μ .

Proof

This follows from the fact that every locally countable collection is σ -locally countable.

3.4. Proposition [I]

If a tritopological space (X, τ, ρ, μ) is $(\tau - \rho)$ -paracompact with respect to μ then it is $(\tau - \rho)$ -semiparacompact with respect to μ .

3.5. Proposition

If a tritopological space (X, τ, ρ, μ) is $(\tau - \rho)$ -semiparacompact with respect to μ then it is $(\tau - \rho)$ -semipara-Lindelöf with respect to μ .

Proof

This follows from the fact that every σ -locally finite collection is σ -locally countable.

3.6. Corollary

If a tritopological space (X, τ, ρ, μ) is $(\tau - \rho)$ -paracompact with respect to μ then it is $(\tau - \rho)$ -semipara-Lindelöf with respect to μ .

Proof

This follows from Proposition 3.4. and Proposition 3.5.

3.7. Theorem

If a tritopological space (X, τ, ρ, μ) is $(\tau - \rho)$ -semipara-Lindelöf with respect to μ then the τ -closed subspace $(Y, \tau_Y, \rho_Y, \mu_Y)$ is $(\tau_Y - \rho_Y)$ -semipara-Lindelöf with respect to μ_Y .

Proof

Let $\Phi = \{U_\lambda : \lambda \in \Lambda\}$ be a τ_Y -open cover of Y . Since each U_λ is a τ_Y -open subset of Y , there is a τ -open subset V_λ of X such that $U_\lambda = V_\lambda \cap Y$. Let $\Pi = \{V_\lambda : \lambda \in \Lambda\} \cup \{X \setminus Y\}$. Then Π is τ -open cover of X . By hypothesis Π is a ρ -open refinement Ω which is σ -locally countable with respect to μ , hence $\Omega = \bigcup_n \Omega_n$ where each $\Omega_n = \{W_{n\gamma} : \gamma \in \Gamma\}$ is locally countable with respect to μ . Set $\Psi = \bigcup_n \Psi_n$ where each $\Psi_n = \{W_{n\gamma} \cap Y : \gamma \in \Gamma\}$. Then Ψ is ρ -open refinement of Φ which is σ -locally countable with respect to μ .

3.8. Theorem

Let (X, τ, ρ, μ) be a tritopological space, and let $\Sigma = \{X_i : X_i \in \tau \cap \rho \cap \mu\}$ be a partition of X . The space (X, τ, ρ, μ)

is $(\tau - \rho)$ -semipara-Lindelöf with respect to μ if and only if the space $(X, \tau_i, \rho_i, \mu_i)$ is $(\tau_i - \rho_i)$ -para-Lindelöf with respect to μ_i .

Proof

The "only if part ",since $X_i = X / \bigcup_{i \neq j} X_j$ is τ -closed then the subspace $(X, \tau_i, \rho_i, \mu_i)$ is $(\tau_i - \rho_i)$ -semipara-Lindelöf with respect to μ for every i by Theorem 3.7.

The " if part ".Let $\Phi = \{U_\lambda : \lambda \in \Lambda\}$ be a τ -open cover of X . The collection $\Pi = \{U_\lambda \cap X_i : \lambda \in \Lambda\}$ be a τ_i -open cover of X_i for every i . By hypothesis Π has a ρ_i -open refinement $\Psi_i = \{W_{i\lambda} : \lambda \in \Lambda\}$ which is σ -locally countable with respect to μ_i . So $\Psi_i = \bigcup_n \Psi_{in}$ where each $\Psi_{in} = \{W_{in\lambda} : \lambda \in \Lambda\}$ is locally countable with respect to μ . Set $\Omega = \bigcup_n \Omega_n$ where $\Omega_n = \bigcup_{i \in I} \Psi_{in}$. Then Ω is ρ -open refinement of Φ which is σ -locally countable with respect to μ .

3.9.Theorem

If (X, τ, ρ, μ) is $(\tau - \rho)$ -semipara-Lindelöf with respect to μ , then the $\tau - F_\sigma$ subspace $(Y, \tau_Y, \rho_Y, \mu_Y)$ is $(\tau_Y - \rho_Y)$ -semipara-Lindelöf with respect to μ_Y .

Proof

Let $\Phi = \{U_\lambda : \lambda \in \Lambda\}$ be a τ_Y -open cover of Y . Since each U_λ is a τ_Y -open cover of Y , there exists a τ -open subset V_λ of X such that $U_\lambda = V_\lambda \cap Y$ for each fixed λ . In the collection $\Pi_n = \{V_\lambda : \lambda \in \Lambda\} \cup \{X / Y_n\}$ form a τ -open cover of X . By hypothesis Π_n has a ρ -open refinement Ω which is σ -locally countable with respect to μ . Then $\Omega = \bigcup_n \Omega_n$ where each $\Omega_n = \{V_\lambda \cap Y : \lambda \in \Lambda\} \cup \{X / Y_n\}$.

$\Omega_n = \{W_{n\gamma} : \gamma \in \Gamma\}$ is locally countable with respect to μ . For each n , let $\Psi = \bigcup_n \Psi_n$ such that $\Psi_n = \{W_{n\gamma} \cap Y : W_{n\gamma} \cap Y \neq \emptyset, \gamma \in \Gamma\}$. Then Ψ is ρ_Y -open refinement of Φ which is σ -locally countable with respect to μ_Y .

3.10.Theorem

If (X, τ, ρ, μ) is $(\tau - \rho)$ -para-Lindelöf with respect to μ , then the $\tau - F_\sigma$ subspace $(Y, \tau_Y, \rho_Y, \mu_Y)$ is $(\tau_Y - \rho_Y)$ -semipara-Lindelöf with respect to ρ_Y .

Proof

Suppose that Y is $\tau - F_\sigma$ set. Then $Y = \bigcup_n Y_n$, where each Y_n is τ -closed. Let $\Phi = \{U_\lambda : \lambda \in \Lambda\}$ be a τ_Y -open cover of Y . Since each U_λ is a τ_Y -open cover of Y , there exists a τ -open subset V_λ of X such that $U_\lambda = V_\lambda \cap Y$ for each λ . For each fixed n the collection $\Pi_n = \{V_\lambda : \lambda \in \Lambda\} \cup \{X / Y_n\}$ form a τ -open cover of X . By hypothesis Π_n has a ρ -open refinement Ω which is σ -locally countable with respect to μ . Then $\Omega = \{W_{\lambda n} : (\lambda, n) \in \Lambda \times N\}$ is locally countable with respect to ρ . For each n , let $\Psi_n = \{W_{n\gamma} \cap Y : W_{n\gamma} \cap Y \neq \emptyset, \gamma \in \Gamma\}$. Let $\Psi = \bigcup_n \Psi_n$. Then Ψ is ρ_Y -open refinement of Φ which is σ -locally countable with respect to μ_Y .

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