

**REFINEMENT OF DIGITAL
IMAGE USING FOURIER
TRANSFORMATION METHOD**

صور الأقمار الصناعية الرقمية والصور الفوتوغرافية

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الخلاصة:

الطريقة المستخدمة في هذا البحث هي لمعالجة صور الأقمار الصناعية الرقمية، والصور الفوتوغرافية، إضافة إلى الأشكال الهندسية المختلفة التي تكون في المستوى الرمادي التي تحتوي على خطوط أفقية تمثل التشويش Noise، الغرض من هذه الدراسة هو تقليل التشويش أو حذفه. تم تحليل الصور هذه في المجال الترددي باستخدام طريقة تحويل فورير، وفي مرحلة معينة من التحليل نحصل على تحويل فورير المعكوس لغرض الحصول معلومات الصورة الرقمية ثانية.

لغرض حذف التشويش تم تكوين مصفوفة (h) ذات العناصر الأحادية باستخدام الأمر التالي: $h = \text{Ones}(256)$ ذات الأبعاد التي تكون مساوية إلى المصفوفة الرئيسية التي تمثل مصفوفة تحويل فورير ذات الأبعاد 256×256 . بعض عناصر المصفوفة تعطى القيم صفر وتبقى العناصر الأخرى ذات قيمة واحد، كل عناصر المصفوفة تضرب مع العناصر المكافئة لها ضمن طيف فورير حسب الأمر $g = \text{Img_fs} * h$ ناتج عملية الضرب سوف يكون طيف فورير الجديد لذلك سوف تزال المساحة المشوشة بسبب عملية الضرب هذه. التشويش الذي تم إزالته يكون ذو لون اسود بارز ضمن طيف فورير المرئي.

I. Abstract

The method is used here to process a digital satellite images, further a photography digital image and many morphology shapes in grayscale mode with horizontal lines which is formed noises to reduce or delete it. We analyze the images here with the frequency domain using the Fourier transformation, the result can be displayed as image, once the manipulation has been in completion, an inversed Fourier transformation is conducted in order to obtain the image information again.

In order to remove noises, matrix (h) is first made, with ones values for each it's elements using the following command: `h = ones (256)`; whose dimension equals to that of the matrix to save the Fourier transformation result, which is 256 x 256. Once certain part of the matrix has been given the value zero, while the other remain to have the value one, each of the elements of the matrix is multiplied with its respective elements of the image Fourier spectrum with the following command: `g = img_fs.* h`; the multiplication will result in a new Fourier spectrum where the noise area has been removed because it has been multiplied by zero. The removed area is displayed in black color in the visualization of the Fourier spectrum.

II. Introduction

Digital image processing is an important example of applied numerical computing. There are at least the following three reasons why people want to compute with the numbers in an image:

1. **Enhancement:** improving or changing a picture in some way.
2. **Compression:** reducing the storage required, usually to economize on storage or speed up transmission.

- 3. **Recognition:** automatically recognize objects, like faces or missile installations. Clearly, this application is in its early stages.

This assignment will have you computing with the elements (pixels) of digital images.

The method used in digital image processing with Fourier Transformation may in general be classified into two, which are spatial domain and frequency domain methods. In the spatial domain method the processing is conducted by directly manipulating the pixel value of an image, while in the frequency domain method the digital image information is first transformed through Fourier transformation and then the Fourier transformation result is manipulated. Once the manipulation has been in completion, an inversed Fourier transformation is conducted in order to obtain the image information again.

The frequency domain method can be used to solve certain problems that are hard to solve using the spatial domain method [1-5].

III. Fourier Transformation

The Fourier transformation of $f(x)$ is defined as follow:

$$F(u) = \int_{-\infty}^{\infty} f(x) \exp[-j2\pi ux] dx \quad \dots\dots\dots (1)$$

where

$$j = \sqrt{-1}$$

On the contrary, if $F(u)$ has been known, the $f(x)$ can be obtained using the following inverse Fourier transformation:

$$f(x) = \int_{-\infty}^{\infty} F(u) \exp[j2\pi ux] du \quad \dots\dots\dots (2)$$

(56)..... REFINEMENT OF DIGITAL IMAGE

The two equations above are known as Fourier transformation pair. If the $f(x)$ is a real number, the $F(u)$ is usually a complex number that may be further analyzed as follow:

$$F(u) = R(u) + I(u) \dots\dots\dots (3)$$

Where $R(u)$ and $I(u)$ are real components and the $F(u)$ is imaginary component. Also, the equation is frequently written as follow:

$$F(u) = | F(u) | e^{j\phi u} \dots\dots\dots (4)$$

where $| F(u) |$ is the magnitude of the $F(u)$, which results from:

$$| F(u) | = [R^2(u) + I^2(u)]^{1/2} \dots\dots\dots (5)$$

$$\phi(u) = \tan^{-1} [I(u) / R(u)] \dots\dots\dots (6)$$

The magnitude function $| F(u) |$ is also referred to as the Fourier spectrum of the $f(x)$, and the $\phi(u)$ is the phase angle of the $F(u)$.

If the $f(x)$ becomes discrete, the discrete Fourier transformation equation is:

$$f(x) = \frac{1}{N} \sum_{x=0}^{N-1} f(x) \exp [- j2\pi ux / N] \dots\dots\dots (7)$$

And

$$f(x) = \sum_{x=0}^{N-1} F(u) \exp [j2\pi ux / N] \dots\dots\dots (8)$$

Since in the digital image processing the data used is in digital/discrete form, the two equations above can be used to conduct transformation and inverse Fourier transformation [6][7].

To analyze the image of the frequency domain, the Fourier transformation result can be displayed as image, where its intensity is proportional with the magnitude of the | F(u) | or the Fourier spectrum. However, since the *dynamic range* of the Fourier spectrum is usually very wide, before it is displayed as image, it must be transformed into:

$$D(u, v) = c \log (1 + | F(u, v) |) \dots\dots\dots (9)$$

Where c is a constant. Subsequently, it is the D(u, v) value that is displayed as the image. The D(u, v) value will have a narrower *dynamic range* than the | F (u, v) | [8 -10].

IV. Research Methodology

To implement an image processing with the frequency domain method, we write a program using MATLAB. In broad outline, the program will follow the following steps:

1. Opening digital image files.
2. Displaying digital images.
3. Conducting Fourier transformation to images.
4. Displaying the Fourier spectrum of the images.
5. Transforming the Fourier transformation results.
6. Displaying the transformed Fourier spectrum.
7. Conducting inverse Fourier transformation.
8. Displaying the processed digital images.

The MATLAB has provided us with some functions that can directly be used to help accelerate program writing. The

MATLAB functions that are very important in writing the program are among others[10]:

1. IMREAD

It is used to read an image of a file. If the image is of grayscale format, it will produce an array of two dimensions containing the intensity information of the image. It supports BMP, JPEG, TIF, PNG, HDF, PCX, and XWD formats.

2. IMSHOW

It is used to display an image on screen.

3. IMWRITE

It is used to save an image in a file. It is the inverse function of the IMREAD.

4. FFT2

It is used to conduct a Fourier transformation to an array of two dimensions, the resulting is obtained it's also formed an array of two dimensions.

5. FFTSHIFT

It is used to shift the Fourier transformation result that enables and easy to visual analysis of a Fourier spectrum. Since the Fourier spectrum is periodic in nature, the shift does not influence the resulting image as the inverse Fourier transformation is conducted.

6. IFFT2

It is used to conduct inverse Fourier transformation to an array of two dimensions.

The first steps made are to read an image file that will be processed and to save the graylevel information of all its pixels in a matrix. The commands used are:

```
nmfile = 'Sky_1.bmp';  
img = imread(nmfile);
```

Subsequently, a Fourier transformation is conducted and it is continued by shifting the Fourier transformation result that its visualization result is more observable and easier to analyze. Therefore, the commands used are:

```
% Fourier Transformastion with FFT  
img_f = fft2(img);
```

```
img_fs = fftshift(img_f);
```

The command *imshow* is then used to display the image and its Fourier spectrum. However, since the *dynamic range* of the Fourier spectrum is very wide when the Fourier spectrum will be displayed, a process must be followed using the command below.

```
img_spectrum = log(1+abs(img_fs));
```

The process will narrow the *dynamic range* that it can be more clearly displayed on the screen and easier analyzed.

In order to remove noises, matrix (h) are first made, whose dimension equals to that of the matrix to save the Fourier transformation result, which is 256 x 256. The matrix (h) is initialized by giving it the value (1) for each of its elements with the following command:

```
h = ones (256);
```

And then, a part of the matrix that indicates the area where the noises are in the image Fourier spectrum is transformed into zero using the following command:

```
% Choose the frequency area to filter
% to remove noise.
for xi = 1 : 256,
for yi = 1 : 256,
% Noise results from the frequency that forms
% the vertical line in the center of spectrum
% The value of the area will be removed by
% multiplying it with 0.
if (yi > 127) & (yi < 130) & ((xi < 123) | (xi > 134))
h(xi,yi) = 0;
end
end
end
```

The aforementioned commands are valid only for the noise in the image we used. If the noise of the Fourier spectrum of other image is in different area, the values of the commands must be modified according to the noise area of the Fourier spectrum that will be removed.

Once certain part of the matrix has been given the value zero, while the other remain to have the value one, each of

the elements of the matrix is multiplied with its respective elements of the image Fourier spectrum with the following command:

```
g = img_fs.* h;
```

The multiplication will result in a new Fourier spectrum where the noise area has been removed because it has been multiplied by zero. The removed area is displayed by black color in the visualization of the Fourier spectrum.

The next step is to conduct an inverse Fourier transformation of the new Fourier spectrum to obtain the image information again with the following command:

```
result = abs(iff2(g));
```

This command is used for the resulting image information is of the pixel data type of 8-bit or 256 gray level that equals to the original image. Followings are the examples of images with their respective Fourier spectrum.

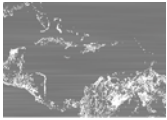

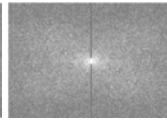
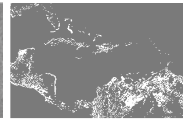

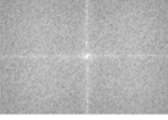
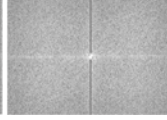


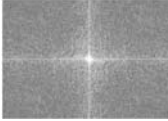
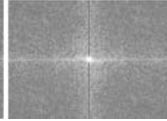


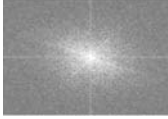
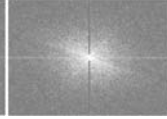

Original Image	Fourier Spectrum	Fourier Spectrum which is	Original Image of IFFT
			
			
			
			

Figure 1: Four satellite images with different horizontal noises.

V. Results and Discussion

We describe an algorithm to digital satellite Image processing using Fourier Transformation method in frequency domain to delete noise as horizontal lines as follow:

1. Initialization
2. Loading of image.
3. Fast Fourier Transformation (FFT) to image.
4. Shifting image.
5. Partition of the working area to four parts, which are named (h_1 , h_2 , h_3 , and h_4) to locate images, such as:
 - a) h_1 to show the original image,
 - b) h_2 to show the Fourier spectrum with dynamic range compression,
 - c) h_3 to show the original image after using inverse Fourier transformation,
 - d) and h_4 to show dynamic range compression from Fourier spectrum.
6. Showing the image.
7. Dynamic range compression to show the Fourier spectrum.
8. Showing the Original Image.
9. Showing spectrum Fourier from this image.
10. Choosing frequency area which will be filtered to delete noise.
11. Noise from the frequency with the shape such as vertical lines.
12. The value in this area will be deleted by multiply by nol.
13. Compression range dynamic from Fourier spectrum to show Fourier spectrum as image.
14. Showing Fourier Spectrum which is filtered.
15. Inverse Fourier Transformation.
16. Show the resulting image.
17. End.

We implement this algorithm by MatLab, since the written program has been tested for some image files with

various kinds of noise as observable in their Fourier spectrums.

Case 1:

In the following images the upper left image is the one with the noise of horizontal lines, while the upper right image is its Fourier spectrum. A vertical line with bright color representing the noise of the image is displayed in the Fourier spectrum.

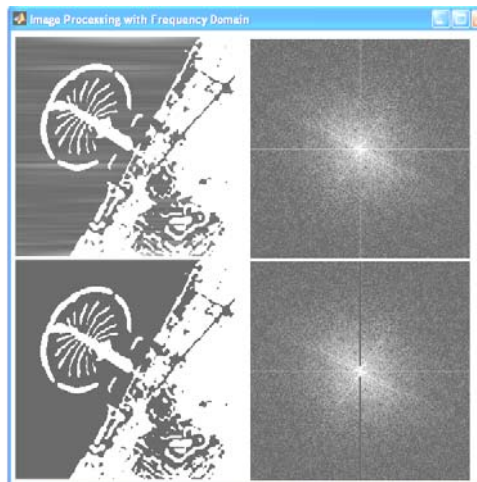


Figure 2: Case study for a digital satellite image.

The lower right image indicates the processed Fourier spectrum image. The process is conducted by multiplying the spectrum value of the chosen area by zero. The chosen area is the part with bright color that forms vertical line because it is this part that represents the noise, while the area in the center of the spectrum is not removed because it contains the information of the image itself. If the area in the center of the spectrum is also multiplied by zero, the resulting image will be dark. Once the chosen area is multiplied by zero, it appears that the cooler of the area turns into dark.

Subsequently, an inverse Fourier transformation is conducted that the image like the lower left one is obtained. It appears that the image becomes clearer because the noise of horizontal lines has been removed.

Case 2:

In this case the principle remains the same and the difference lies in the noise as represented by the presence of vertical bright color of the Fourier spectrum and the image.

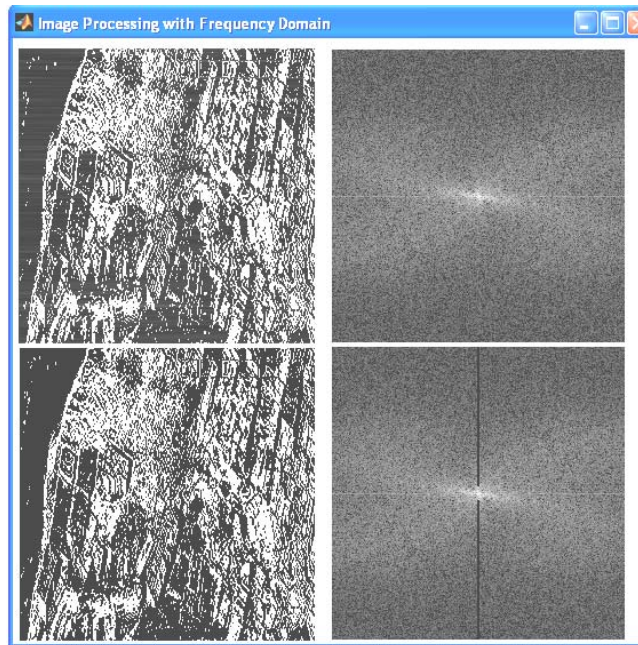


Figure 3: Case study for another digital satellite image

Once the chosen area of the spectrum has been multiplied by zero, an inverse Fourier transformation is conducted and it appears that the resulting image becomes much clearer than before.

It is clearly observed from the two cases above that the digital image processing with frequency domain method refines the images with certain noises.

Such noises that can be removed or reduced using this method are the ones that produce certain pattern in their Fourier spectrum such as straight line. If the noise is

distributed and has irregular form, it will be difficult to choose the area causing the noise.

VI. Conclusion

From the two cases study and many other examples of digital image which is presented here, and when we applied the Fourier transformation method with frequency domain, we conclude that, this method can be used to solve certain problems that are hard to solve using another method such as spatial domain method. The digital image information obtained here by transformed first using Fourier transformation and then the Fourier transformation result is manipulated. Once the manipulation has been in completion, an inverse Fourier transformation is conducted to obtain the image information again. The noises that can be removed or reduced using this method are the ones that produce certain pattern in their Fourier spectrum such as straight lines, which is clearly can be seen from the resulting images. Generally, the written program has functioned well to manipulate these digital images by deleting the noises that will be chosen in an interactive manner.

VIII. References

- [1] J. S., Robert, 1989, "Digital Image Processing and Computer Vision", John Wiley & Sons.
- [2] R.C. Gonzalez, E. W., Richard, 1993, "Digital Image Processing", Addison-Wesley Publishing Company Inc., USA.
- [3] J. C., Russ, 1995, "The Image Processing Handbook", CRC Press.
- [4] C., Anderson and M. D., Dahleh, 1996, "Rapid Computation of the Discrete Fourier Transform", SIAM J. Sci. Comput., 17(4): 913 - 919.
- [5] S. J. Sangwine and T. A. Ell. 1998, "The Discrete Fourier Transform of a Color Image", In J. M. Blackledge, editor, Second IMA Conference on Image Processing: 22-25.
- [6] S., Kristian, 2001, "An overview of Fourier analysis for signal processing", Dept. of Applied Mathematics, University of Colorado at Boulder.

<http://amath.colorado.edu/courses/3310/2001fall/Improc/Webpages/fourier/fourier.scc>.

- [7] J. M., Blackledge 2005, “Digital Image Processing: Mathematical and Computational Methods”, Horwood Publishing Chichester, West Sussex
- [8] F., Brackx, , N., De Schepper, F., Sommen, 2006, “The Two-Dimensional Clifford-Fourier Transform”, J Math Imaging Vol. (26), 5–18.
- [9] L. Ying and S. Fomel, 2008, “Fast Computation of Partial Fourier Transforms”, Technical Report, University of Texas at Austin.
- [10] User’s Guide, 1993, “Image Processing Toolbox for Use with MATLAB”, Version 2, The Math works.