

Design and Analysis of Cosine Modulated Filter banks and Modified DFT Filter Banks

**تصميم وتحليل أجراء مرشحات معدلة بدالة جيب التمام وأجراء
مرشحات بدالة تحويل فوريير مجزئة ومعدلة**

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Abstract:-

In this paper, an iterative algorithm to design near perfect reconstruction (PR) of Cosine Modulated Filter banks is presented, and then improved to design Modified discrete Fourier transform (MDFT) Filter Banks. Design examples are given for both filter banks with comparison between them.

Keywords: Filter Banks, Iterative Algorithm, Cosine Modulated Filter banks, Modified discrete Fourier transform Filter Banks.

1. Introduction

The multicarrier system is very important modulation technique which provides high resistance against intersymbol interference under high bit rate transmission condition. Orthogonal frequency division multiplexing (OFDM) system is a very popular version from multicarrier communication systems which is used in many modern communication standards because it can be implement efficiently and easily using fast Fourier transform (FFT) with a simple equalizer. This simplicity holds as long as the time guard interval (CP), which is inserted in front of transmit OFDM symbol, is greater than impulse response of the

channel [1]. However, insertion of CP will reduce bandwidth efficiency. Another drawback of OFDM is the considerable overlap between adjacent subcarriers, which will make the spectral containment very poor and make the OFDM system very sensitive to impairments such as narrow band interference [2]. Another drawback of the OFDM system is that the high peak to average power ratio (PAPR) which means the need to use a linear amplifier with large dynamic range to avoid the distortion in peaks [3]. Keeping in mind that if the system has a high PAPR this will make the implementation of the amplifier very difficult with low power efficiency.

The filter bank based multicarrier (FBMC) system is another architecture of multicarrier communication system which is proposed by many authors to solve the weak points in conventional OFDM system. However, the full development of FBMC system has not been considered sufficiently yet [3]. Existing solutions include wavelet packet schemes, such as the ones found in [4], and offset QAM-based OFDM (OFDM-OQAM) systems [5]. Another very promising option, which has been the focus of many recent publications, is the modulated filter bank-based transceiver [6]. In this regard, both the cosine function [7], and the complex exponential [8], via the DFT or the FFT, can be employed [6]. In this paper, the cosine modulated filter bank (CMFB) and the modified DFT (MDFT) filter bank will be studied and their design and performance analysis are given.

The rest of this paper organized as follows: section 2 discussed an efficient algorithm for the design of M-channel cosine-modulated near PR filter banks. Some design examples of M-channel cosine-modulated near PR filter banks are in section 3. Section 4 presents the design of M-channel MDFT filter banks. Some design examples are presented in section 5. Finally, Conclusion will be presented in section 6, where our results will

be summarized and a comparison between the two types of filter banks will be stated.

2. Design of M-Channel near PR CMFB.

Modulated filter banks is one of the important filter structures that involve the ease of design and implementation [9] with very fast algorithm [10]. An important subclass of modulation filter banks is the cosine modulated filter banks [11]. The analysis and synthesis filters are cosine-modulated versions of a single prototype filter. In this way, two advantages can be obtained. First, the system offers high computational efficiency. Furthermore, modulation can be done by fast techniques such as fast discrete cosine transforms (DCT). Second, the system will provide high performance filter design [11].

Princen and Bradley [12], and Vetterli and Le Gall [13] have shown that an M-channel PR modulated filter bank can be obtained with FIR analysis and synthesis filters as long as their impulse responses have a length $N = KM$ [10], where K is an even number. The fact that the length of filter is greater than the number of subchannels does not cause a decrease in the achievable throughput, since each filter is orthogonal to itself and all others at shifts of LM , where L is any integer, so the transmitted waveforms can be overlapped for maximum throughput [14].

The main challenge in the design of this class of filter bank is to eliminate the amplitude distortion from the frequency response of the prototype filter which can be done by using nonlinear optimization techniques, especially for the design of perfect reconstruction (PR) filter banks, which may lead to further difficulties such as non-convergence and step-size selection [9]. For practical applications with lossy channel coding and quantization, the PR property is desirable but not necessary. In addition, the PR filter bank with high stopband attenuation is generally difficult to achieve [9].

An iterative algorithm for the design of multi-channel cosine modulated quadrature mirror filter (QMF) banks with near perfect reconstruction is proposed by Xu et al. [7]. He proposed a new optimization algorithm in which the objective function is converted into a quadratic function of the filter coefficients whose minimum point could be obtained analytically.

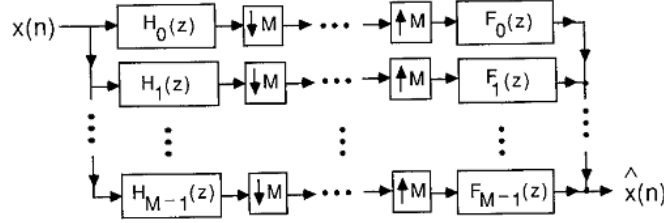


Figure 1: M-channel filter banks

This algorithm can be derived if consider the cosine modulated QMF banks shown in Fig.1, the transfer functions of the analysis and synthesis filters can be described by $H_k(z)$ and $F_k(z)$ for $k=0,1,\dots, M-1$, respectively, where M represent the number of channels in the filter banks. All these filters are obtained by modulating the transfer function $P(z)$ of a prototype linear phase lowpass FIR filter with cutoff frequency $\pi/2M$, as [9]:

$$H_k(z) = a_k c_k P(z W_{2M}^{((k+1)/2)}) + a_k^* c_k^* P(z W_{2M}^{-((k+1)/2)}) \quad (1)$$

$$F_k(z) = a_k^* c_k P(z W_{2M}^{((k+1)/2)}) + a_k c_k^* P(z W_{2M}^{-((k+1)/2)}) \quad (2)$$

For $0 \leq k \leq M-1$, where $a_k = e^{j\theta_k}$, $c_k = W_{2M}^{(k+\frac{1}{2})(\frac{N-1}{2})}$, $W_{2M} = e^{-j\pi/M}$ and N is the length of prototype filter $P(z)$ which is chosen as $N=2KM$ where K is positive integer number. It can be shown that the impulse responses of the analysis and synthesis filter $H_k(z)$ and $F_k(z)$ are given by [9]:

$$h_k(n) = 2p(n)\cos\left[\frac{(2k+1)\pi}{2M}\left(n - \frac{N-1}{2}\right) + \theta_k\right]$$

$$f_k(n) = 2p(n)\cos\left[\frac{(2k+1)\pi}{2M}\left(n - \frac{N-1}{2}\right) - \theta_k\right]$$

Where $p(n)$ is the impulse response of the prototype filter $P(z)$. As shown by Vaidyanathan in [11], θ_k can be chosen as $\theta_k = (2k + 1)\pi/4$, for $0 \leq k \leq M - 1$, to cancel significant aliasing terms and ensure a relatively flat overall amplitude response. The overall frequency response of the filter bank can be written as [7]

$$T_o(e^{j\omega}) = \frac{e^{-j\omega(N-1)}}{M} \sum_{k=0}^{M-1} |H_k(e^{j\omega})|^2 \quad (5)$$

Equation (5) shows that there is no phase distortion due to the using of linear phase prototype filter. Now, the amplitude distortion can be eliminated by using an optimization process to ensure $\sum_{k=0}^{M-1} |H_k(e^{j\omega})|^2 = 1$ for $0 \leq \omega \leq \pi$, then perfect reconstruction will be achieved and the reconstruction delay will be $N-1$ sampling periods.

It can be shown that $|T_o(e^{j\omega})|$ has a period of π/M and if the prototype filter $P(z)$ has a large stopband attenuation, then eqn. 5 gives:

$$T_o(e^{j\omega}) \approx \frac{e^{-j\omega(N-1)}}{M} \times \left[\left| P(e^{j(\omega - \frac{k\pi}{M} - \frac{\pi}{2M})}) \right|^2 + \left| P(e^{j(\omega - \frac{(k+1)\pi}{M} - \frac{\pi}{2M})}) \right|^2 \right] \quad (6)$$

For ω in the range of $(k + \frac{1}{2})\pi/M \leq \omega \leq (k + \frac{3}{2})\pi/M$. If the quantity in the square brackets in eqn.6 made constant over $[0, \frac{\pi}{M}]$ then $|T_o(e^{j\omega})|$ will be constant for all frequencies [12], which means the amplitude distortion is eliminated. Now, the design problem can be reduced to the case of minimizing the objective function

$$E = E_1 + \alpha E_2 \quad (7a)$$

where

$$E_1 = \int_0^{\frac{\pi}{M}} \left[M_p^2(\omega) + M_p^2\left(\omega - \frac{\pi}{M}\right) - 1 \right]^2 d\omega$$

$$E_2 = \int_{\omega_s}^{\frac{\pi}{M}} M_p^2(\omega) d\omega \quad (7c)$$

$$M_p(\omega) = 2p^T c(\omega) \quad (7d)$$

$$c(\omega) = [\cos\left(\frac{(N-1)\omega}{2}\right) \dots \cos\left(\frac{\omega}{2}\right)]^T \quad (7e)$$

$$p = [p(0) p(1) \dots p\left(\frac{N}{2} - 1\right)]^T \quad (7f)$$

E_1 deals with the amplitude distortion of the overall transfer function $T_o(z)$ and E_2 deals with the stopband attenuation of the prototype filter $P(z)$, where $\omega_s = \frac{\pi}{2M} + \varepsilon$, and ε is a positive constant which depends on the required transition width. Parameter α is a positive weight.

Using standard optimization method to minimizing E in eqn.7a tends to be time-consuming since the objective function is highly nonlinear [7]. Instead of minimizing function E directly, Xu et al. in [7] adopt an iteration procedure in which the objective function in eqn.7a is modified to

$$\vec{E} = \vec{E}_1 + \alpha \vec{E}_2 \quad (8a)$$

$$\begin{aligned} \vec{E}_1 = \sum_{0 \leq \omega \leq \frac{\pi}{M}} [& M_p(\omega) M_q(\omega) \\ & + M_p\left(\omega - \frac{\pi}{M}\right) M_q\left(\omega - \frac{\pi}{M}\right) - 1]^2 \end{aligned} \quad (8b)$$

$$\begin{aligned} \vec{E}_2 = \int_{\omega_s}^{\pi} M_q^2(\omega) d\omega &= 4q^T \left[\int_{\omega_s}^{\pi} c(\omega) c^T(\omega) d\omega \right] \\ &= q^T U_s c(w) \end{aligned} \quad (8c)$$

$$M_q(\omega) = 2q^T c(\omega) \quad (8d)$$

$$q = [q(0) q(1) \dots q\left(\frac{N}{2} - 1\right)]^T \quad (8e)$$

At the beginning of the iterative procedure, filter $P(z)$ is first designed using one of the conventional methods such as the

window method. The summation in eqn. 8b is carried out over a set of sampling points $\Omega_p = [\omega_{p1} = 0, \omega_{p1}, \dots, \omega_{pk} = \frac{\pi}{M}]$. Then $\hat{E}$ in eqn. 8a can be formulated as a quadratic function of q given by

$$\hat{E} = (U_q - d)^T (U_q - d) + \alpha(q^T U_s q) \quad (9)$$

Where d is a column vector with each entry being a 1, and

$$U = H(\Omega_p)U_t(\Omega_p) + H\left(\Omega_p - \frac{\pi}{M}\right)U_t\left(\Omega_p - \frac{\pi}{M}\right) \quad (10)$$

$$H(\Omega_p) = \text{diag}[M_p(\omega_{p1}), M_p(\omega_{p2}), \dots, M_p(\omega_{pk})] \quad (11)$$

$$U_t(\Omega_p) = 2 \begin{bmatrix} \cos\left(\frac{(N-1)\omega_{p1}}{2}\right) & \dots & \cos\left(\frac{\omega_{p1}}{2}\right) \\ \cos\left(\frac{(N-1)\omega_{p2}}{2}\right) & \dots & \cos\left(\frac{\omega_{p2}}{2}\right) \\ \vdots & \dots & \vdots \\ \cos\left(\frac{(N-1)\omega_{pk}}{2}\right) & \dots & \cos\left(\frac{\omega_{pk}}{2}\right) \end{bmatrix} \quad (12)$$

The (i,j) th entry of U_s is given by

$$u_{ij}^{(s)} = 4 \begin{cases} \frac{\pi - \omega_s}{2} - \frac{\sin[(2i-N-1)\omega_s]}{2(2i-N-1)} & \text{for } i \neq j \\ \frac{\sin[(i-j)\omega_s]}{2(j-i)} - \frac{\sin[(i+j-N-1)\omega_s]}{2(i+j-N-1)} & \text{for } i = j \end{cases} \quad (13)$$

For $i, j=1, 2, \dots, N/2$. Since $U^T U + \alpha U_s$ is positive definite, $\hat{E}$ has a global minimum point at

$$q = (U^T U + \alpha U_s)^{-1} \cdot (U^T d) \quad (14)$$

Having obtained q , a linear formula is used to update p as

$$p = (1 - \tau)p + \tau q \quad (15)$$

Where τ , $0 < \tau < 1$, is a smoothing parameter. The above process is repeated until $\|p - q\|_2$ reach a specified tolerance.

The iterative algorithm can now be summarized in terms of the following steps:

1. Design a linear phase lowpass FIR filter of length N with cutoff frequency $\frac{\pi}{2M}$ using conventional method. Then make use of the coefficient vector of the filter obtained to initialize p .
2. Use eqn. 13 to compute matrix U_s .
3. Form matrix U and vector q by eqn.10 and eqn. 14, respectively.

If $\|p - q\|_2 < \varepsilon$, where ε is a predescribed tolerance, output p as the design result and stop; otherwise update p using eqn. 15 with τ in the range $0 < \tau < 1$, say $\tau = 0.5$, and repeat from step 3.

3. Design Examples

In this section, the above algorithm was applied to design filter banks with $M=2, 4, 16$, and 64 . The design was implemented using MATLAB with the parameters shown in Table 1. Figure 2 shows the frequency response of the analysis filter bank of the design examples. The total response of analysis filter banks in case of $M=2$ and 64 are shown in Figure 3, the amplitude distortion is decreasing with increasing M and N , this can be explained if we bring in mind that the approximation done based on the large stopband attenuation conditions, this means large value of N and then large value of M .

4. Design of Modified DFT filter bank

The DFT filter bank is the easiest filter banks to implement and it has the simplest structure, which is shown in Fig. 4. Among general M -channel multirate filter banks, DFT filter banks are of highest computational efficiency [8]. Unlike cosine modulated filter banks, they cannot cancel alias components caused by decimation of subband signals [8]. However, Karp and Fliege [15] proposed a modified DFT (MDFT) filter banks which is able to cancel adjacent channel alias spectra resulting in almost PR. Compared to cosine modulated filter banks, the

MDFT filter banks offers many advantages such as highest computational efficiency, complex valued signal processing with a different mapping into the subbands and low propagation delays [15].

The M-channel MDFT filter bank, as shown in Fig. 5, is a critically sampled complex-modulated filter bank. Its number of channels M is even [8]. All the analysis filters $H_k(z)$ and synthesis filters $F_k(z)$ are derived by modulating an N-length zero-phase prototype filter $P(z)$, whose cut-off frequency is π/M .

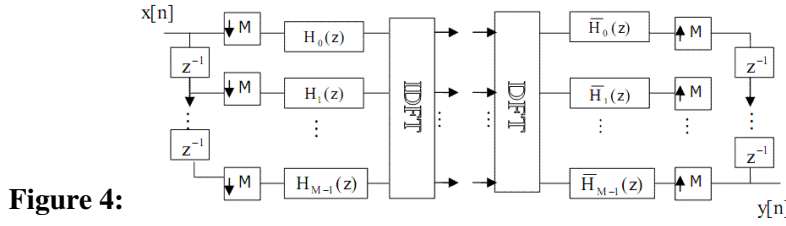


Figure 4:

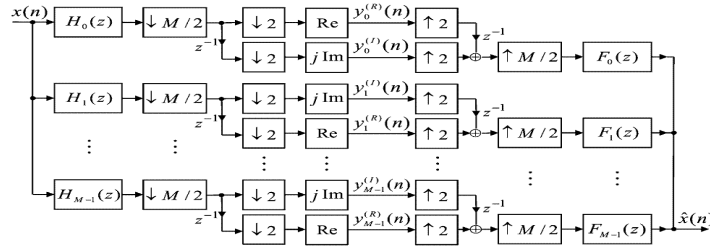


Figure 5: Modified DFT filter bank.

$$H_k(z) = F_k(z) = z^{-(N-1)/2} P(zW_M^k) \quad (16)$$

For $0 \leq k \leq M - 1$, where W_M is defined as $W_M = e^{-j2\pi/M}$. In the z domain, the input signal $X(z)$ and the output signal $\hat{X}(z)$ of the filter bank are related by

$$\begin{aligned} \hat{X}(z) &= \frac{z^{-\frac{M}{2}}}{M} \sum_{i=0}^{\frac{M}{2}-1} \sum_{k=0}^{M-1} F_k(z) H_k(zW_M^{2l}) X(zW_M^{2l}) \\ &= T_0(z)X(z) + \sum_{l=1}^{\frac{M}{2}-1} T_l(z)X(zW_M^{2l}) \end{aligned} \quad (17)$$

Where

$$T_l(z) = \frac{z^{-\frac{M}{2}}}{M} \sum_{k=0}^{M-1} F_k(z) H_k(zW_M^{2l}),$$

$$0 \leq l \leq \frac{M}{2} - 1 \quad (18)$$

In the above expressions, $T_0(z)$ is called the distortion transfer function and determines the distortion caused by the overall system for the unaliased component $X(z)$ of the input signal. The remaining transfer functions $T_l(z)$, $1 \leq l \leq M/2-1$, are called the alias transfer functions and determined how well the aliased components $X(zW_M^{2l})$ of the input signal are attenuated.

The design method consists of optimizing this function to minimize the stopband attenuation of the filter [16]. The optimization could be time consuming since the cost function (stopband attenuation of the filters) is a highly nonlinear function with respect to this function. Consequently, linear phase paraunitary filter banks, where the filters have good stopband attenuation, are difficult to obtain [16]. However, in the previous section, a highly efficient design method have been presented for high-resolution (complexity) cosine modulated filter banks, this algorithm will be improved here to design MDFT filter banks.

The design algorithm of cosine-modulated filter banks (CMFB) discussed in the section 2 can be used to design MDFT filter banks with decimation the design prototype filter with a factor 2 and scaled by $\sqrt{2}$ [15]. This algorithm can be summarized by following steps:

1. Chose the number of the channels of MDFT filter banks (M_{MDFT}) which equal to half the number of channels of CMFB (M_{CMFB}).
2. Design a prototype of CMFB as discussed in section 2 with number of subchannel M_{CMFB} and order N_{CMFB} .
3. Now, the prototype of MDFT filter banks is decimation in factor 2 from the prototype design in step 3 and then scaled by $\sqrt{2}$.

5. Design Examples

In this section the above algorithm was applied to design filter banks with $M=2, 4, 16$, and 64 . The design was implemented using MATLAB with the parameters shown in Table 2. Fig. 6 shows the frequency response of the analysis filter bank of the design examples. The total response of analysis filter banks in case of $M=2$ and 64 are shown in Fig. 7, the behavior of filter banks is similar to that design in CMFB because of the similarity in algorithms of the design. However, it can be seen that the performance of MDFT filter banks is better than the performance of CMFB with low computational cost.

6. Conclusion

The performance characteristics of the CMFB and MDFT filter bank have been presented. As shown in previous sections, there are slight differences in the efficiency of the filter operations with the complex signal. The MDFT filter banks designed to process complex signal, and, CMFB designed to process the real signal only, however, one can use CMFB with complex signal by using two filter banks one for the real part of the signal and the other to the imaginary part of it. Therefore, cosine-modulated filter bank needs the highest number of operations, while, MDFT filter bank with complex signal processing needs always somewhat less operations. Also, there is a difference in the kind of mapping: the cosine modulated filter bank maps the real part of the input signal into the real parts of the subband signals. The corresponding holds for the imaginary parts. The MDFT filter bank maps the real part of the input signal into the real as well as into the imaginary parts of the subband signals. The same holds for the imaginary part. This fact might be exploited to eliminate correlated signal components in coding applications.

Major advantage of the MDFT filter banks structures is that it consists of multiplexers, demultiplexers, FIR filters, and FFT or

IFFT blocks where all these parts are very regular and hence suitable for designing efficient integrated circuit architecture.

5. References

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الملخص:-

تصميم وتحليل أجراء مرشحات معدلة بدالة جيب التمام وأجراء مرشحات بدالة تحويلية فوريير مجزئة ومعدلة

في هذا البحث تم تقديم خوارزمية تكرارية لتصميم أجراء مرشحات معدلة بدالة جيب التمام وتعطي تقريباً إعادة بناء مثالية، ومن ثم تم تطوير الخوارزمية لتصميم أجراء مرشحات معدلة بدالة تحويلية فوريير المجزئة. ولقد تم تقديم عدد من الأمثلة لكلا النوعين إضافة لمناقشة النتائج والمقارنة.

Table 1: Parameters used in the design examples

parameters	M=2	M=4	M=16	M=64
N	60	60	120	320
τ	0.5	0.5	0.5	0.5

α	100	100	100	100
ε	10^{-4}	10^{-4}	10^{-4}	10^{-4}

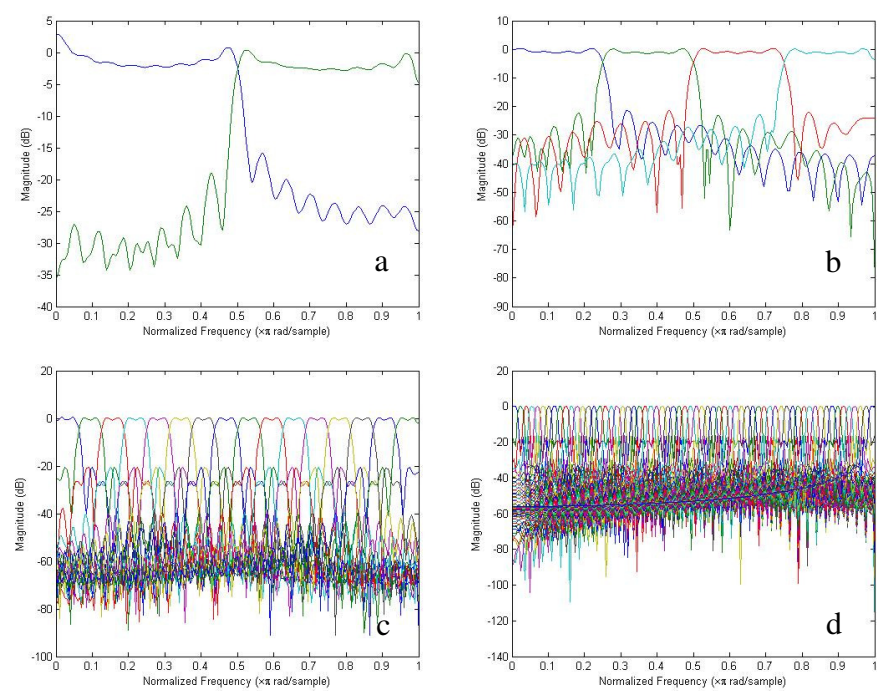


Figure 2: The frequency response of the analysis filter banks of a) 2-channel filter bank, b) 4-channel filter banks, c) 16-channel filter banks , and d) 64-channel filter banks.

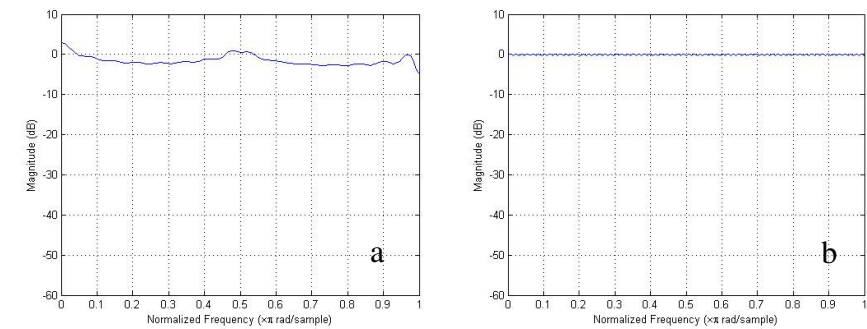


Figure 3: The total frequency response of analysis filter banks of a) 2-channel filters banks, and b) 64-channel filter banks.

Table 2: Parameters used in the design examples.

parameters	$M_{\text{MDFT}}=2$	$M_{\text{MDFT}}=4$	$M_{\text{MDFT}}=16$	$M_{\text{MDFT}}=64$
N_{MDFT}	60	80	192	576
τ	0.5	0.5	0.5	0.5
α	100	100	100	100
ε	10^{-4}	10^{-4}	10^{-4}	10^{-4}

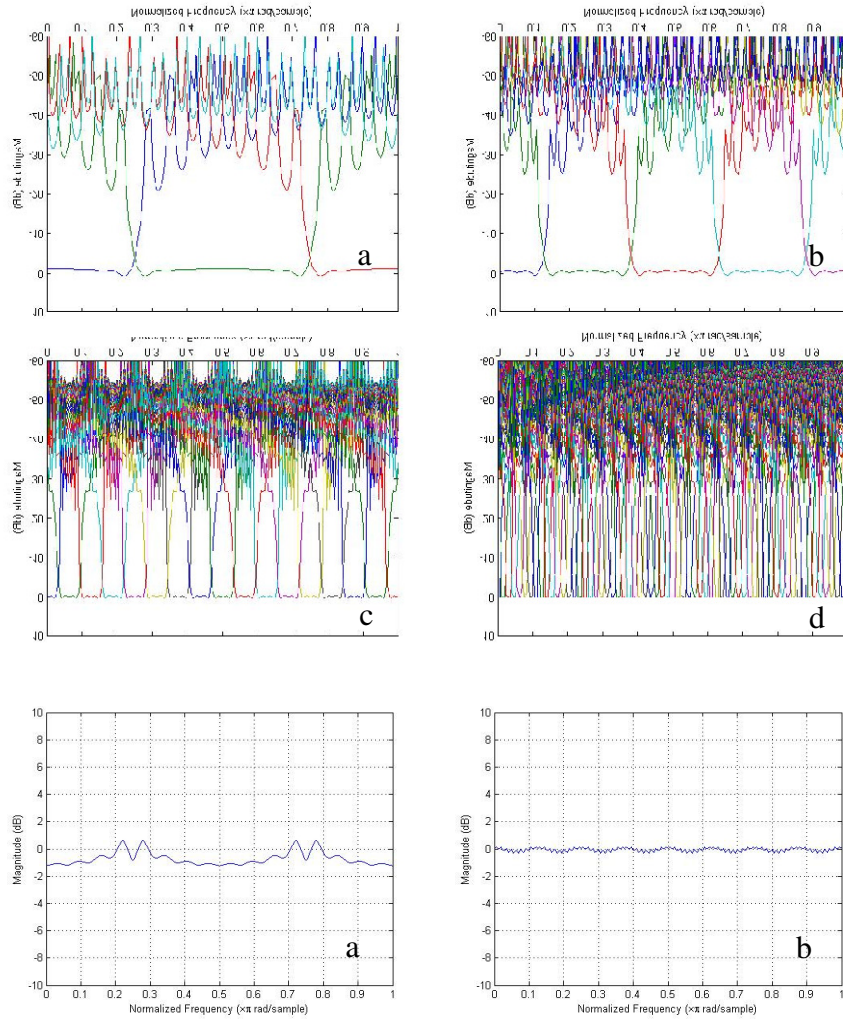


Figure 6: The frequency response of the analysis filter banks of a) 2-channel filter bank, b) 4-channel filter banks, c) 16-channel filter banks, and d) 64-channel filter banks.