Linear Quadratic Regulator Design using Particle Swarm Optimization

تصميم المنظم الخطي التربيعي باستخدام أمثلية الحشد الجزيئي

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Abstract:

This paper presents the design of Linear Quadratic Regulator using Particle Swarm Optimization (PSO) method. The PSO method is used to set the elements of the weighting matrices subject to a proposed cost function. The proposed cost function is a combination of the quadratic performance index and integral time square error. The proposed design can overcome the difficulty in setting the weighting matrices with the suitable elements. To show the effectiveness of the proposed controller, a controlled vibration absorber is considered as a case study.

Keywords: LQR, Optimal control, PSO, Vibration absorber.

1. Introduction

The Linear Quadratic Regulator (LQR) is widely used in many applications and has received considerable attention since 1950. It is an optimal multivariable feedback control approach that minimizes the excursion in state trajectories of a system with minimum control effort. LQR is a method of reducing the performance index to a minimum value which achieves an acceptable performance of the system [1, 2]. In a variety of references, the effectiveness of LQR has been described (see, for example [3] and [4]). Furthermore, some of new control algorithms in the literature have been derived based on LQR like the state derivative feedback in [5] and LQR for nonlinear control systems in [6].

In this work, the design of LQR controller using PSO method is proposed. The Particle Swarm Optimization (PSO) method is used to automate and simplify the design procedure of LQR technique and obtain the optimal state feedback gains. The proposed method overcomes the drawbacks of the classic LQR technique and enhances the accuracy and performance of the system.

2. Linear Quadratic Regulator Design

For a linear time invariant system with

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t) \tag{1}$$

where $x(t)$ represents the state vector, $y(t)$ represents the output and $u(t)$ represents the control vector. The LQR approach constructs a linear state feedback law expressed by [7]:

$$u(t) = -Kx(t) \tag{2}$$

where K represents the state feedback gain matrix. The design of K is a tradeoff between the transient response and the control effort. The optimal control approach to this design tradeoff is to search for the control vector in equation (2) which minimizes the following performance index:

$$J = \int_0^\infty \left(x^T(t)Qx(t) + u^T(t)Ru(t)\right)dt \tag{3}$$

where $Q \in Q^{n\times n}$ represents symmetric positive semidefinite state weighting matrix, and $R \in R^{m\times m}$ is a symmetric positive definite control weighting matrix. The control gain matrix is given by:

$$K = R^{-1}B^TP \tag{4}$$

where P is the unique symmetric positive semi-definite solution to the algebraic *Riccati* equation (ARE) which expressed by:

$$PA + A^TP + Q - PBR^{-1}B^TP = 0 \tag{5}$$

It is required that (A, B) is stabilizable and (A, C) is detectable.

In this work, the PSO method is used to obtain the optimal values of the weighting matrices of LQR. The PSO concept involves, at each time step, changing the velocity of each particle towards its global best and local best locations. The particles are manipulated according to the following equations of motion [8, 9]:

$$v_i^{k+1} = w \times v_i^k + c_1 \times rand \times (x_i^b - x_i^k) + c_2 \times rand \times (x_i^g - x_i^k) \quad (6)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad (7)$$

where v_i^k is the particle velocity, x_i^k is the current particle position, w is the inertia weight, x_i^b and x_i^g are the best value and the global best value, *rand* is a random function between 0 and 1, c_1 and c_2 are learning factors. The PSO requires only a few lines of computer code to realize PSO algorithm. Also it is a simple concept, easy to implement, and computationally efficient algorithm [10].

The weighting matrices Q and R are important components of LQR and their elements have great effects on the system performance. The selection of elements of these matrices is normally based on iterative procedure using experience and physical understanding of the problems involved [7]. Commonly, a trial and error method is used to set the elements of Q and R matrices. The trial and error method is very simple but it is not feasible for high dimensional systems and even for simple systems is labor-intensive, time consuming approach and the expected performance cannot be guaranteed [1]. Therefore, to enhance the LQR design, the PSO method is used to overcome the difficulty in finding the right weighting matrices which limits the application of LQR subject to the following proposed cost function:

$$J = \int_0^{\infty} \left(x^T(t) Q x(t) + u^T(t) R u(t) \right) dt + \int_0^{t_f} t e^2(t) dt \quad (8)$$

where t_f is the estimated settling time.

3. Numerical Example

A vibration absorber system is considered in this work to show the effectiveness of the presented method. The configuration of the system is shown in Figure 1. This system is MIMO linear system with two inputs and two outputs. The dynamic equations of this system are [11]:

$$m_1\ddot{y}_1(t) + b_1\big(\dot{y}_1(t) - \dot{y}_2(t)\big) + k_1 y_1(t) = u_1(t)$$

$$m_2\ddot{y}_2(t) + b_1\big(\dot{y}_2(t) - \dot{y}_1(t)\big) + k_2 y_2(t) = u_2(t) \qquad (9)$$

The state space form is:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1}{m_1} & \frac{-b_1}{m_1} & 0 & \frac{b_1}{m_1} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{b_1}{m_2} & \frac{-k_2}{m_2} & \frac{-b_1}{m_2} \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \qquad (10)$$

where $x_1(t) = y_1(t), x_2(t) = \dot{y}_1(t), x_3(t) = y_2(t), x_4(t) = \dot{y}_2(t)$.

The nominal values of system parameters are: $m_1 = 10\ kg, m_2 = 30\ kg, k_1 = 2.5\frac{KN}{m}, k_2 = 1.5\frac{KN}{m}\ and\ b_1 = 30\frac{Ns}{m}$.

For the considered (2×2) system, the proposed cost function in equation (8) is rewritten as:

$$J_{min} = \int_0^{\infty}\big(x^T(t)Qx(t) + u^T(t)Ru(t)\big)dt + \int_0^{t_f} t(e_1^2(t)+e_2^2(t))dt \qquad (11)$$

Figure 2 shows the transient response of the uncontrolled system. It is shown that the system needs more than 50 *sec*. to reach the steady state. To improve the system performance, the LQR with tuning of Q and R matrices using PSO method subject to the proposed cost function in equation (11) was first applied. The resulting transient response is shown in Figure 3. It is clear that the trajectory of the controlled system approaches the equilibrium within 0.2 *sec*. for the first output and 1.4 *sec*. for the second output. The obtained control inputs using LQR are shown in Figure 4. The PSO parameters are set to: swarm size=25, $c_1 = c_2 = 2$, *w*=1.5 and number of iterations=2000. The convergence rate of PSO algorithm

$$Q = \begin{bmatrix} 332.8379 & 0 & 0 & 0 \\ 0 & 0.1018 & 0 & 0 \\ 0 & 0 & 912.4203 & 0 \\ 0 & 0 & 0 & 80.9421 \end{bmatrix}, R = \begin{bmatrix} 7.1054 \times 10^{-6} & 0 \\ 0 & 7.3616 \times 10^{-5} \end{bmatrix}$$

$$K = \begin{bmatrix} 6.8051 \times 10^3 & 366.7954 & 1.1705 \times 10^3 & 385.3612 \\ -219.6383 & 12.3984 & 3.5002 \times 10^3 & 1.1094 \times 10^3 \end{bmatrix}$$

4. Conclusion

In this work the design of Linear Quadratic Regulator using PSO method has been presented. The PSO method was used to optimal tuning of LQR weighting matrices subject to a proposed cost function. The proposed cost function has combined the quadratic performance index and integral time square error performance index. The results showed that the proposed design can achieve a desirable time response specifications with automatic and optimal tuning of LQR weighting matrices.

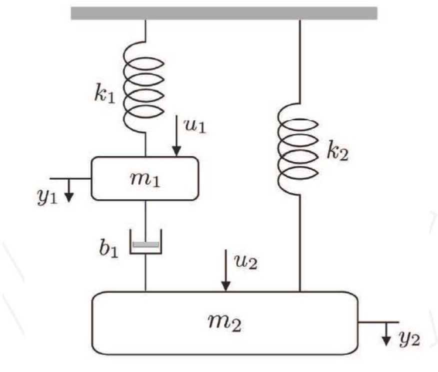

Figure 1: Vibration absorber system

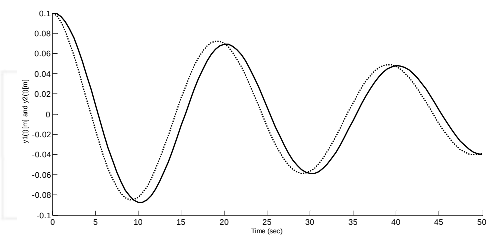

Figure 2: Transient response of the uncontrolled system with $x(0) = [0.1 \quad 0 \quad 0.1 \quad 0]^T$.

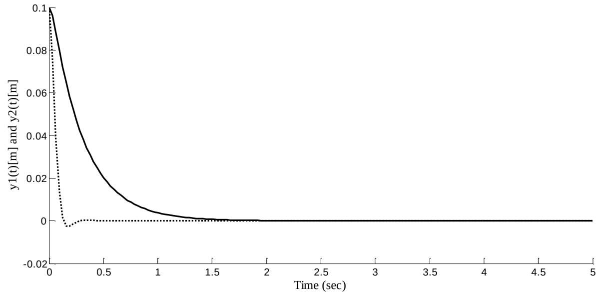

Figure 3: Transient response of the controlled system using LQR with $x(0) = [\mathbf{0.1} \quad \mathbf{0} \quad \mathbf{0.1} \quad \mathbf{0}]^T$

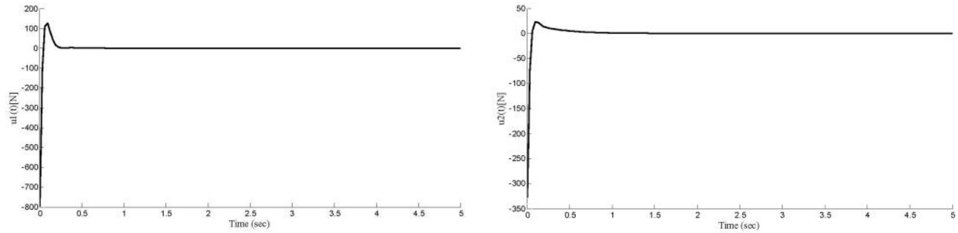

Figure 4: Control inputs of the controlled system using LQR.

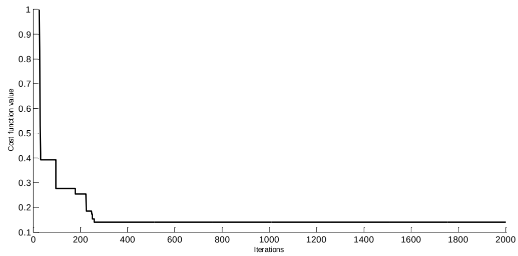

Figure 5: PSO algorithm convergence rate for LQR.

الخلاصة:

هذا البحث يقدم تصميم للمنظم الخطي التربيعي باستخدام أمثلية الحشد الجزيئي. ان نظرية الحشد الجزيئي تستخدم لضبط عناصر مصفوفات الوزن اعتمادا على دالة كلفة مقترحة والتي هي عبارة عن تشكيلة من معامل الاداء التربيعي والتكامل لمربع الخطأ. ان التصميم المقترح يتغلب على الصعوبة في عملية اختيار عناصر مصفوفات الوزن ويجد القيم المناسبة لتلك العناصر. ان فعالية التصميم المقترح تم اثباتها عن طريق تطبيق المنظم على منظومة امتصاص الاهتزازات.

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